WSB

SAMPLE EXAM

Financial Risk Management

Time allowed: 90 min

Answer Question 1 in Section 1 and 2 questions from Section 2. Section 1 is worth 60% of the final mark. Marks assigned to specific sub sections are displayed in brackets. Each question in Section 2 is worth 20% of the final mark.

Notes are not permitted in this examination.

Section 1

Question 1

Consider an American put option with time to expiry 1 year, and a strike of 100. The current price of the underlying is 100. Divide the time to expiry into two 6-months intervals. Assume that in each 6-month interval, the price can either rise by 10, or fall by 10, with unknown probability. The risk-free (continuously compounding) rate is 0.05.

(a) Using a binomial tree, identify the circumstances under which early exercise would be rational for the holder of this option. [Hint: you need to use either no–arbitrage argument or risk neutral valuation] [20]

(b) What is the value of the option? [10]

(c) What would the value of this option be if it were European and not American? Explain briefly why your answer should be no greater than that obtained in (b). [10]

(d) How would your answers in (b) and (c) change if the underlying asset paid dividend of 5 in 5 months' time? [20]

Section 2

Question 1

Consider European call option and European put option written on the same underlying stock, with the same strike price, K, and time to expiry τ. The risk-free rate is r. The current price of the underlying stock is S_t. Let the values of the two options be c and p, respectively.

Prove that, in the absence of arbitrage opportunities:

$$c + Ke^{-r\tau} = p + S_t$$

What is the name given to this equation?

Question 2

Consider a long forward contract to purchase a non-dividend paying stock in 4 months. Assume that the current stock price is £90 and the 4-month risk-free interest rate is 6% per annum.

(a) What is the non-arbitrage price?

(b) What sort of arbitrage trade would be possible if the forward price was £95, and how much profit would be made?

Question 3

Suppose that a bond with maturity value m = £100 pays a coupon of c = £5 for three years (τ = 3). Assume that the yield is y = 4%. Find:

(a) The price of the bond.

(b) Macauley Duration

Refer to the bond described above as bond 1, and assume that another bond (bond 2) is also available with m = £100, c = £5, τ = 6, y = 3%.

(c) A company has signed a contract to pay £1m to a client in four years' time. In what combination should the company purchase the two bonds in order to eliminate interest rate risk?

Financial Risk Management - Formulae

Bonds

Price of a coupon-paying bond:

$$p = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \ldots + \frac{c+m}{(1+y)^\tau}$$

Macaulay Duration:

$$D = -\left(\frac{1+y}{p}\right)\frac{\partial p}{\partial y} = \frac{1}{p}\left[\frac{1\times c}{(1+y)} + \frac{2\times c}{(1+y)^2} + \frac{3\times c}{(1+y)^3} + \ldots + \frac{\tau\times(c+m)}{(1+y)^\tau}\right]$$

Options

The Black-Scholes formula for the value, c, at time t, of a European call option on a non-dividend-paying stock is:

$$c = S_t \Phi(d_1) - \exp(-r\tau) K \Phi(d_2)$$

where:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}$$

and:

S_t is the price of the underlying stock at time t,

$\tau\ (= T - t)$ is the time to expiry,

r is the continuously compounded risk-free rate of interest,

σ is the price volatility of the underlying stock,

K is the strike price,

$\Phi(.)$ is the cumulative distribution function for a standard normal random variable.

Table 1: The standard normal distribution function

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5	0.504	0.508	0.512	0.516	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.758	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.998	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.999	0.999
3.1	0.999	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Procedure for finding $\Phi(z) = P(Z < z)$:

Look down the first column for the first decimal place of *z*, look along the top row for the second decimal place of *z*, and read $\Phi(z)$ from the middle of the table.