Time Value of Money

Question 8:

A company is planning to set aside money to repay $100 million in bonds that will be coming due in 10 years. If the appropriate discount rate is 9%,

a. how much money would the company need to set aside at the end of each year for the next 10 years to be able to repay the bonds when they come due?

Answer:

Use annuity give PV formula:

$$Annuity\ given\ Present\ Value = PV\left[\frac{r}{1-\frac{1}{(1+r)^n}}\right]$$

PV of 100 m to be set aside in 10 years' time is:

100/(1.09)^10=42.24108 (million $)

Thus every year we need to set aside:

42.24(0.09/(1-(1/(1.09^10))))=6.582 (million $)

b. how would your answer change if the money were set aside at the beginning of each year?

Answer:

Since we set aside the money at the end of each period, we need to set aside a lower amount:

6.582/1.09=6.03854 (million $)

Question 9:

What is the value of 15-year corporate bonds, with a coupon rate of 9%, if current interest rates on similar bonds is 8%? How much would the value change if interest rates increased to 10%? Under what conditions will this bond trade at par (face value)?

Answer:

Let's assume the par value is 1000.

The value of the bond is:

=(90/0.08)*(1-(1/(1+0.08)^15))+1000/(1.08^15)=1085.595

If the interest rate change to 10% the value would be:

=(90/0.10)*(1-(1/(1+0.10)^15))+1000/(1.10^15)=923.9392

The interest rate would have to be equal to 9% in order for the bond to trade at par (i.e. for the PV of the future cash flows of the bond to be equal to 1000)

(90/0.09)*(1-(1/(1+0.09)^15))+1000/(1.09^15)=1000

Question 10:

What is the value of stock in a company that currently pays out $1.50 per share in dividends and expects these dividends to grow 6% a year forever? (You can assume that investors require a 13% return on stocks of equivalent risk.)

Answer:

Use growing annuity formula:

=1.5*(1.06)/(0.13-0.06)=22.71429

Question 11:

What is the value of stock in a company that currently pays out $1.00 per share in dividends, and expects these dividends to grow 15% a year for the next 5 years, and 6% a year forever after that? (You can assume that investors require a 12.5% return on stocks of equivalent risk and that the dividend payout ratio will double after the fifth year.)

Answer:

Dividend in year 10: 1*(1.15^10)=4.04558

PV of 1st 10 dividends: growing annuity formula

=1*((1.15)/(0.125-0.15))*(1-((1.15^10)/(1.125^10)))=11.30744

PV of dividends after year 10 (in terms of money of year 10):

=4.04558*(1.06)/(0.125-0.06)=65.97407

Value of the stock:

=11.30744+65.97407/(1.125^10)=31.6239

Question 12:

You buy a 10-year zero-coupon bond, with a face value of $1000, for $300. What is the rate of return you will make on this bond?

Answer:

Solve following for r:

300=1000/(1+r)^10

r is approx. equal to 12.79% because 1000/(1.1279)^10=300.1194

Question 13:

You are reviewing an advertisement by a finance company offering loans at an annual percentage rate of 9%. If the interest is compounded weekly, what is the effective interest rate on this loan?

Answer:

Assume 52 weeks in a year

=(1+0.09/52)^52-1=0.094089

Question 14:

You have a relative who has accumulated savings of $ 250,000 over his working lifetime and now plans to retire. Assuming that he wishes to withdraw equal instalments from these savings for the next 25 years of this life, how much will each instalment amount to if he is earning 5% on his savings?

Answer:

Assume 25 annual instalments, solve for CF

250000=(CF/0.05)*(1-1/(1.05^25))

CF=250000*0.05/(1-1/(1.05^25))=17738.11

Question 15:

You are offered a special set of annuities by your insurance company, whereby you will receive $20,000 a year for the next 10 years and $30,000 a year for the following 10 years. How much would you be willing to pay for these annuities, if your discount rate is 9% and the annuities are paid at the end of each year? How much would you be willing to pay if they were at the beginning of each year?

Answer:

If they are paid at the end of the year:

PV of the 20k annuity:

=(20000/0.09)*(1-(1/(1.09^10)))=128 353.2

PV of the 30k annuity (in terms of money in year 10)

=(30000/0.09)*(1-(1/(1.09^10)))=192 529.7

Fair price of the annuity:

=128353.2+192529.7/(1.09^10)=209 679.8

If they are paid at the beginning of the year:

If we get 20000 at the beginning of the year then at the end of the year it will grow to 20000*1.09

=(20000*(1.09)/0.09)*(1-(1/(1.09^10)))+ (30000*(1.09)/0.09)*(1-(1/(1.09^10)))/(1.09^10)=228 551

Question 16:

A bill that is designed to reduce the nation's budget deficit passes both houses of legislature. Congress tells us that the bill will reduce the deficit by $500 billion over 10 years. What it does not tell us is the timing of the reductions.

Year	*Deficit Reduction*
1	$ 25 Billion
2	$ 30 Billion
3	$ 35 Billion
4	$ 40 Billion
5	$ 45 Billion
6	$ 55 Billion
7	$ 60 Billion
8	$ 65 Billion

If the federal government can borrow at 8%, what is the true deficit reduction in the bill?

Answer:

Calculate PV of the cash flows

=25/1.08+30/1.08^2+35/1.08^3+40/1.08^4+45/1.08^5+55/1.08^6+60/1.08^7+65/1.08^8=241.4661 ($ billion)

Question 17:

New York State has a pension fund liability of $25 billion, due in 10 years. Each year the legislature is supposed to set aside an annuity to arrive at this future value. This annuity is based on what the legislature believes it can earn on this money.

a. Estimate the annuity needed each year for the next 10 years, assuming that the interest rate that can be earned on the money is 6%.

Answer:

Since we set aside the money

=25*(0.06/(1-(1/(1.06^10))))=3.396699

b. The legislature changes the investment rate to 8% and recalculates the annuity needed to arrive at the future value. It claims the difference as budget savings this year. Do you agree?

Answer:

=25*(0.08/(1-(1/(1.08^10))))=3.725737

I don’t agree.

Question 18:

Mr X is making the following amounts for the next 5 years:

Year	*Amount*
0 (now)	$ 5.5 million (Sign up Bonus)
1	$ 4 million
2	$ 4 million
3	$ 4 million
4	$ 4 million
5	$ 7 million

a. Assuming that Mr X can make 7% on his investments, what is the present value of his contract?

Answer:

Let's assume the numbers in the table are the nominal figures, thus the PV is:

=5.5+4/1.07+4/1.07^2+4/1.07^3+4/1.07^4+7/1.07^5=24.03975

b. If you wanted to raise the nominal value of his contract to $30 million, while preserving the present value, how would you do it? (You can adjust only the sign up bonus and the final year's cash flow.)

Answer:

=5.5+4*4+7=28.5

We need to raise the nominal amount by 1.5m

=1.8+4/1.07+4/1.07^2+4/1.07^3+4/1.07^4+12.2/1.07^5=24.04728

=1.8+4*4+12.2=30

Question 20:

You bought a house a year ago for $250,000, borrowing $200,000 at 10% on a 30-year term loan (with monthly payments). Interest rates have since come down to 9%. You can refinance your mortgage at this rate, with a closing cost that will be 3% of the loan. Your opportunity cost is 8%. Ignore tax effects.

a. How much are your monthly payments on your current loan (at 10%)?

Answer:

=(200000*(0.1/12))/(1-(1/(1+(0.1/12))^360))=1755.143

b. How would your monthly payments be if you could refinance your mortgage at 9% (with a 30-year term loan)?

Answer:

In the 1st month we paid 1755.143 out of which were interest1666.667, and 88.47647 capital repayment

=200000*(0.1/12)=1666.667

IN the 2nd month we paid 1755.143 out of which 1665.929 was interest and 89.21364 capital repayment

After 1 year the outstanding loan is 198985.2

If we want to refinance the loan, and we assume we have 29 years left our monthly payments would be:

=(198985.2*(0.09/12))/(1-(1/(1+(0.09/12))^348))=1612.094

c. You plan to stay in this house for the next 5 years. Given the refinancing cost (3% of the loan), would you refinance this loan?

Answer:

Every month for the next 5 years we will save

=1755.143-1612.094=143.049

PV (in terms of money today) of the 143.049 received monthly for next 5 years is

=(143.049/(0.09/12))*(1-1/(1+(0.09/12))^(5*12))=6891.153

Whereas the cost of refinancing is:

=198985.2*0.03=5969.556

Thus it would make sense to refinance.

d. How much would interest rates have to go down before it would make sense to refinance this loan (assuming that you are going to stay in the house for five years)?

Answer:

It already make sense.

Question 21:

You are 35 years old today and are considering your retirement needs. You expect to retire at age 65 and your actuarial tables suggest that you will live to be 100. You want to move to the Bahamas when you retire. You estimate that it will cost you $ 300,000 to make the move (on your 65th birthday) and that your living expenses will be $30,000 a year (starting at the end of year 66 and continuing through the end of year 100) after that.

a. How much will you need to have saved by your retirement date to be able to afford this course of action?

Answer:

Assume 5% cost of capital

=300000+(30000/0.05)*(1-1/(1.05^35))=791 225.8

b. You already have $50,000 in savings. If you can invest money, tax-free, at 8% a year, how much would you need to save each year for the next 30 years to be able to afford this retirement plan?

Answer:

PV of the cost at 8% rate is:

=791225.8/(1.08^30)=78629.91

You already have 50 000, thus you need to save PV equal to 28 629.91 of through next 30 years

=78629.91-50000=28 629.91

Which means a year an annual saving of2542.121

=(28629.91*0.08)/(1-(1/1.08^30))=2542.121

c. If you did not have any current savings and do not expect to be able to start saving money for the next 5 years, how much would you have to set aside each year after that to be able to afford this retirement plan?

Answer:

The PV of the saving in terms of money in 5 years' time is:

=791225.8/(1.08^25)=115533.1

You will need to set aside 10 823 a year for the 25 years to be able to cover your spending:

=(115533.1*0.08)/(1-(1/1.08^25))=10823

Question 22:

You have been hired to run a pension fund for TelDet Inc, a small manufacturing firm. The firm currently has $5 million in the fund and expects to have cash inflows of $2 million a year for the first 5 years followed by cash outflows of $ 3 million a year for the next 5 years. Assume that interest rates are at 8%.

a. How much money will be left in the fund at the end of the tenth year?

Answer:

=5+((2/0.08)*(1-1/(1.08^5)))-((3/0.08)*(1-1/1.08^5))/(1.08^5))=4.833306

b. If you were required to pay a perpetuity after the tenth year (starting in year 11 and going through infinity) out of the balance left in the pension fund, how much could you afford to pay?

Answer:

=4.833306*0.08=0.386664(m$)

Question 23:

You are an investment advisor who has been approached by a client for help on his financial strategy. He has $250,000 in savings in the bank. He is 55 years old and expects to work for 10 more years, making $100,000 a year. (He expects to make a return of 5% on his investments for the foreseeable future. You can ignore taxes)

a. Once he retires 10 years from now, he would like to be able to withdraw $80,000 a year for the following 25 years (his actuary tells him he will live to be ninety years old.). How much would he need in the bank 10 years from now to be able to do this?

Answer:

=(80000/0.05)*(1-1/(1.05^25))=1 127 516

b. How much of his income would he need to save each year for the next 10 years to be able to afford these planned withdrawals ($80,000 a year) after the tenth year?

Answer:

Over next 10 year he needs to save the PV of:

=1127516/(1.05^10)-250000=442 197

An annuity with PV of 441 197 assumes annual saving of

=(D44*0.05)/(1-(1/1.05^10))=57 266.54

c. Assume that interest rates decline to 4% 10 years from now. How much, if any, would you client have to lower his annual withdrawal by, assuming that he still plans to withdraw cash each year for the next 25 years?

Answer:

We assume he has 1 127 516 in the bank in 10 years' time, thus he can withdraw 7 2174.48 a year for the next 25 years.

=(B44*0.04)/(1-(1/1.04^25))=7 2174.48

Question 24:

You have been asked to estimate the value of a 10-year bond with a coupon that will be low initially but it is expected to grow later in the bonds life. The coupon is expected to be 5% of the face value of the bond (which is $ 1000) for the first 5 years, and will increase by 1% every year for the next 5 years ñ the coupon rate will be 6% in year 6, 7% in year 7, 8% in year 8, 9% in year 9 and 10% in year 10. Estimate the value of this bond. Cost of capital is 5%.

Answer:

=50/1.05+50/1.05^2+50/1.05^3+50/1.05^4+50/1.05^5+60/1.05^6+70/1.05^6+80/1.05^8+90/1.05^9+1100/1.05^10=1100.948

Question 25:

You are trying to assess the value of a small retail store that is up for sale. The store generated a cash flow to its owner of $ 100,000 in the most year of operation, and is expected to have growth of about 5% a year in perpetuity.

If the rate of return required on this store is 10%, what would your assessment be of the value of the store?

What would the growth rate need to be to justify a price of $ 2.5 million for this store?

Answer:

The value of the store:

=100000*(1.05)/(0.1-0.05)=2 100 000

The growth rate would need to be around 5.78%:

=100000*(1.0578)/(0.1-0.0578)=2 506 635

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