Bonds…

Question 1

Current prices of three zero-coupon bonds, all with maturity value 100, and with different times to maturity (τ) are shown in the following table:

τ	p
1	98
2	95
3	90

Compute the yield to maturity (y) for each of the three bonds. Hence sketch the yield curve. What does the shape of this particular yield curve tell us about current market expectations?

Answer: We've seen that par yield of a bond -- coupon paying or otherwise -- is the coupon rate (c/m). Therefore, par yield for a zero-coupon bond = 0%.

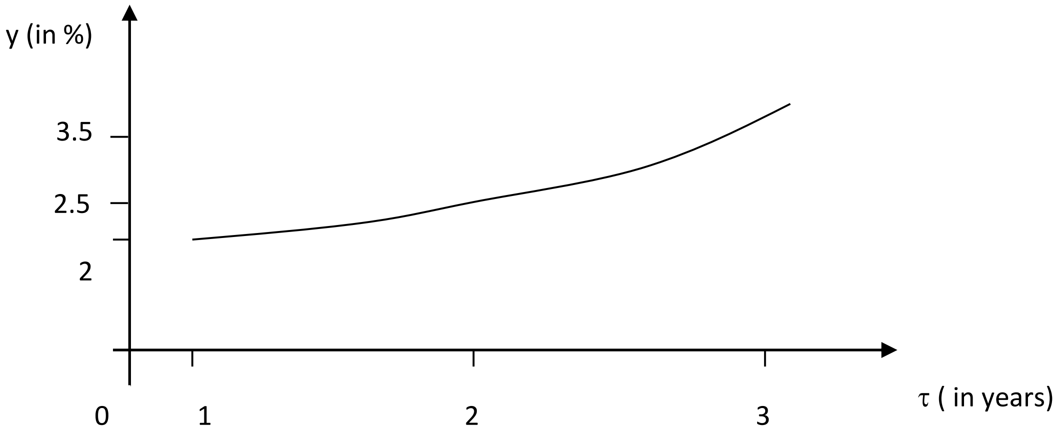
Answer:

$$y_1 = \left(\frac{100}{98}\right)^{\frac{1}{1}} - 1 = 0.020408163$$

$$y_2 = \left(\frac{100}{95}\right)^{\frac{1}{2}} - 1 = 0.025978352$$

$$y_3 = \left(\frac{100}{90}\right)^{\frac{1}{3}} - 1 = 0.035744169$$

See the yield curve below:



The positive slope means that investors expect interest rates to rise in the future.

Question 2

Suppose that a bond with maturity value m = £100 pays a coupon of c = £5 for three years (τ = 3). Assume that the yield is y = 4%. Find:

(i) The price of the bond.

(ii) Macauley Duration

Obtain the answers using a hand calculator and then using Excel.
The price of the bond is:

$$p = \frac{5}{1+0.04} + \frac{5}{(1+0.04)^2} + \frac{5+100}{(1+0.04)^3} = \underline{102.775}$$

and the Macauley Duration is:

$$D = \frac{1}{102.775}\left[\frac{1\times 5}{1+0.04} + \frac{2\times 5}{(1+0.04)^2} + \frac{3\times(5+100)}{(1+0.04)^3}\right] = \underline{2.861}$$

Note that you can use the Excel sheet “bonds” to work out these. The three important functions are:

=PRICE(settlement date, maturity date, coupon rate, yield, maturity value, frequency, basis)

=YIELD(settlement date, maturity date, coupon rate, price, maturity value, frequency, basis)

=DURATION(settlement date, maturity date, coupon rate, yield, frequency, basis)

We normally set frequency to 1 (although most real bonds have frequency=2) and basis to 1.

For the dates: in this example, you could use 30/12/2009 for the settlement date, and 30/12/2012 for the maturity date, since this would give the required time to maturity of 3 years.

Question 3

Refer to the bond of Question 2 as bond 1, and assume that another bond (bond 2) is also available with m = £100, c = £5, τ = 6, y = 3%.

A company has signed a contract to pay £1m to a client in four years’ time. In what combination should the company purchase the two bonds in order to eliminate interest rate risk?

We require a portfolio of the two bonds that has overall duration 4. The durations of the individual bonds are seen to be 2.861 and 5.368. Let w and 1-w be the weights of bonds 1 and 2 in the portfolio. We require:

$$2.861w + 5.368(1-w) = 4$$
$$\therefore 5.368 - 4 = (5.368 - 2.861)w$$
$$\therefore w = \frac{1.368}{2.507} = \underline{0.546}$$

In order to eliminate interest rate risk, the company needs to combine bonds 1 and 2 in the ratio:

0.545 : 0.455

Futures and Forwards

Question 1
Suppose that zero interest rates with continuous compounding are as follows:

Maturity (years)	Rate (% per annum)
1	2
2	3
3	3.7
4	4.2
5	4.5

Calculate forward interest rates for the second, third, fourth and fifth years.

Answer:
Year 2: 4%
Year 3: 5.1%
Year 4: 5.7%
Year 5: 5.7%

Question 2

Consider a long forward contract to purchase a non-dividend paying stock in 3 months. Assume that the current stock price is £100 and the 3-month risk-free interest rate is 5% per annum. Show that the no-arbitrage forward price is £101.2578

Answer:
F=S*exp(rτ)=100*exp(0.05*0.25)=101.2578

What sort of arbitrage trade would be possible if the forward price was £103, and how much profit would be made?

Answer:
If the forward price is relatively high, then the arbitrageur can borrow £100 at the risk-free rate of 5% per annum, buy one share, and short a forward contract to sell one share in 3 months. At the end of the 3 months, he delivers the share and receives £103. The sum of money required to pay off the loan is equal to:

100*exp(0.05*0.25)=101.2578

Thus the arbitrageur locks in the profit of 103-101.2578=1.742155 (£) at the end of 3 months.

Question 3
What is the current value of the forward contract described in question 2 if the forward price is £103?

Answer:

It is equal to the present value of the profit calculated above:

1.742*exp(-0.05*0.25)=1.72

Suppose you purchase the forward contract for £103. After 3 months, the stock price has risen to £110. What is the value of the forward contract now?

Answer:
Assuming that the future stock price is £110, the value of the forward contract today is:

(110-103)*exp(-0.05*0.25)=6.913

Question 4

The two-month interest rates in Switzerland and the US are 2% and 5% per annum, respectively, with continuous compounding. The spot price of the Swiss franc is $0.8. The futures price for a contract deliverable in two month is $0.81. What arbitrage opportunities does this create?

Answer:
The theoretical futures rice is:

0.8*exp((0.05-0.02)*2/12)=0.804

The actual futures price is too high. This suggest that an arbitrageur should buy Swiss francs and short Swiss francs futures.

Question 5

The standard deviation of monthly changes in the spot price of live cattle is 1.2 (in cents per pound). The standard deviation of monthly changes in futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200 000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40 000Pounds of cattle. What strategy should the beef producer follow?

Answer:
The optimal hedge ratio is:
H=0.7*1.2/1.4=0.6
The beef producer requires a long position in 200 000* 0.6= 120 000 lbs of cattle, therefore he should take long position in 3 December contracts closing out position on Nov 15.

European Options

Question 1: Binary Options
The current price of a stock is £40. The “volatility” of the returns on the stock is σ = 0.15 . The risk-free rate of interest is 6% per annum.

(a) What is the value of a binary call option on the stock, with a strike price of £50, with expiry in 9 months’ time, and with a payoff of £1?

Answer:

Binary Call: K=50; τ=9/12; Q=1

Value of the option:

$$c = Q\exp(-r\tau)\Phi(d_2)$$

where:

$$d_2 \equiv \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

Inserting the numbers, we have:

$$d_2 = \frac{\ln\left(\frac{40}{50}\right) + \left(0.06 - \frac{0.15^2}{2}\right)\left(\frac{9}{12}\right)}{0.15\sqrt{\frac{9}{12}}} = \frac{-0.2231 + 0.0366}{0.1299} = -1.4357$$

$$\Rightarrow c = 1 * \exp\left(-0.06 * \frac{9}{12}\right)\Phi(-1.4357) = 0.9560 * 0.0755 = 0.0722$$

The call option is only worth 7p.

(b) What is the value of a binary put option on the stock, with a strike price of £50, with expiry in 9 months’ time, and with a payoff of £1?

Answer:

Binary Put: K=50; τ=9/12; Q=1

$$p = Q\exp(-r\tau)\Phi(-d_2)$$

where d_2 is defined as in (a). Inserting the numbers, we have:

$$p = 1 * \exp\left(-0.06 * \frac{9}{12}\right)\Phi(1.4357) = 0.9560 * 0.9245 = 0.8838$$

The put option is worth 88p.

(c) If you hold one of each of the options described in (a) and (b), what is the value of your portfolio?

Answer:

Assume the market prices are equal to the valuations computed above. Purchasing one call and one put will cost 0.0722 + 0.8838 = 0.9560. This is the (current) value of your portfolio.

Question 2: The Binomial Model

A European Call Option has time to expiry one year ($\tau = 1$). Divide this time to expiry into TWO intervals of equal length. Assume that in either of the two intervals, the underlying stock price can either rise by 20 units or fall by 15 units with equal probabilities. The current stock price is $S_t = 100$. The risk-free rate of interest is $r = 0.05$ (assume simple compounding).

(a) Create a table showing all of the possible values of the stock price at expiry S_T, and the probability of each.

(b) Find the value of the call option if the strike price is $K = 100$.

(c) Find the value of the call option if the strike price is $K = 120$.

(d) What would be the answer in (b) if we use continuous compounding?

(e) Assume that the probabilities are unknown. Use risk neutral valuation or no-arbitrage argument to value the option.

Answer: See the Excel file available on the module website.

Question 3

EMI stock is currently trading at 148p. Consider European call and put options due to expire in 25 days, with strike price 150p, at a risk-free rate of 6%, and with volatility (σ) estimated to be 0.04.

(a) Programme the Black-Scholes formulae in Excel and calculate the value of the two options.

(b) If the current price of the call is 1.25p, what is the implied volatility?

(c) If the current price of the put is 2.62p, what is the implied volatility?

Answer: See the Excel file available on the module website.

Question 4

Assume that $K=100$, $S_t =110$, $\tau = 0.5$ (i.e. time to expiry is 6 months), and the risk-free rate is 0.05. The current price of the put option is $p = 2.00$.

(a)What would the price of the call option (c) need to be for put-call parity to hold?

Answer:
The price of the call option (c) needs to be:

$$c = p + S_t - K\exp(-r\tau) =$$
$$= 2 + 110 - 100 * \exp(-0.05 * 0.5) = 14.469$$

(b)If the price of the call option is 5.00, describe the arbitrage that would be possible, and calculate the profit that would result.

Answer:
Portfolio (1) consisting of put and a stock is overpriced in relation to portfolio (2) consisting of call and bond, therefore the arbitrageur should buy securities in portfolio (2) and short sell those in portfolio (1). This strategy involves buying the call and short selling both the put and the stock, which will generate upfront positive cash flow of:

-5+ 2 + 110 = 107

When invested at risk free rate this cash flow will bring:

$$107 * \exp(0.05 * 0.5) = 109.7087$$

in 6 months. If the stock price at the expiration of the option is greater than 100, the call will be exercised; if the price is less than 100, the put will be exercised. In either cases the arbitrageur will end up buying one share for 100. This share can be used to close the short position, thus the net profit is equal to:

109.7087 – 100 = 9.7087

The Greeks…

Question 1:
Consider a European Call Option with a strike of 110. The current price is 100, and the time to expiry is 3 months. The current market price of the option is 1.34. The risk-free rate is 0.05.

(a) Find the implied volatility of the underlying.

(b) The results of your econometric model gives you reason to believe that the volatility of the underlying will be 0.25 over the next 3-month period. Is the option described above over-priced or under-priced?

(c) If you purchase 1000 options now, how many units of the underlying do you need to short-sell in order to construct a "perfect hedge"?

(d) Which of the two possible measures of volatility have you used in obtaining your answer to (c)?

(e) After one month, you review your position. The price of the underlying has fallen from 100 to 98. What do you need to do in order to "re-hedge".

(f) Look at "gamma". Is re-hedging more or less important than it was at the time the option was purchased?

Answer: See the Excel file.

VAR

Question 1

Investor X has a position in Asset A of £20 000 and of £10 000 in Asset B. If the daily volatilities of both assets are 1% and correlation coefficient between their returns is 0.5. What is 3-day 99% VaR of Investor X portfolio? [N(-2.33)=Φ(-2.33)=0.01]

Answer:

The standard deviation of the daily change in the investment in asset A is £200 and of asset B is£100. The variance of the portfolio's daily change:

$200^2+100^2+2*0.5*200*100=$ 40 000+ 10 000+ 10 000= 60 000

Therefore the standard deviation of portfolio's daily changes is equal to £244.95, whereas the standard deviation of 3-day change is:

244.95*√3 = 424.26

As N(-2.33)=0.01, the 3-day 99 percent value at risk is therefore:

2.33 * 424.26 = 988.52

Question 2

Financial institution owns a portfolio of options on the US dollar-sterling exchange rate. The delta of the portfolio is 56. The current exchange rate is 1.5000. Derive an approximate linear relationship between the change in the portfolio value and the percentage change in the exchange rate. If the daily volatility of the exchange rate is 0.7%, estimate the 10day 99% VaR.

Answer:

The approximate relationship between the daily change in the portfolio value, ΔP , and the daily change in the exchange rate, ΔS, is

$$\Delta P = 56\Delta S$$

The percentage daily change in the exchange rate, Δx , equals ΔS/1.5 . It follows that

$$\Delta P = 56 * 1.5\Delta x$$

or

$$\Delta P = 84\Delta x$$

The standard deviation of Δx equals the daily volatility of the exchange rate, or 0.7 percent. The standard deviation of ΔP is therefore 84 * 0.007 = 0.588. It follows that the 10-day 99 percent VaR for the portfolio is:

$$0.588 * 2.33 * \sqrt{10} = 4.33$$

Suppose you know that the gamma of the portfolio in Question 5 is 16.2. How does this change your estimate of the relationship between the change in the portfolio value and the percentage change in the exchange rate?

Answer:

The relationship is

$$\Delta P = 56 * 1.5 \Delta x + \frac{1}{2} * 1.5^2 * 16.2 * \Delta x^2$$

or

$$\Delta P = 84 \Delta x + 18.225 \Delta x^2$$

American Options & Dividends

Question 1

Consider an American put option with time to expiry 1 year, and a strike of 110. The current price of the underlying is 100. Divide the time to expiry into four 3-month intervals. Assume that in any 3-month interval, the price can either rise by 10, or fall by 10. The risk-free rate is 0.08.

Under what circumstances would early exercise be rational for the holder of this option? What is the value of the option?

Answer: see the Excel file

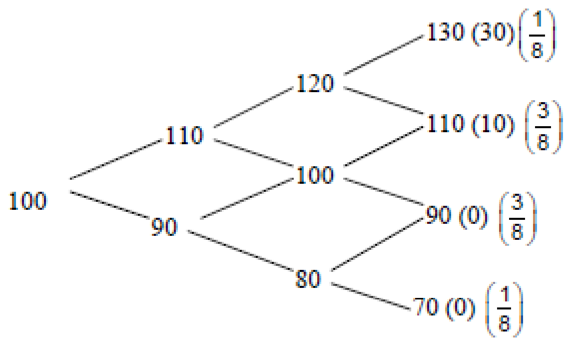
Question 2

Consider a European call option with time to expiry 1 year, and a strike of 100. The current price of the underlying was 100. Divide the time to expiry into three 4-month intervals, and assume that in any 4-month interval, the price can either rise by 10, or fall by 10, with equal probability. The risk-free rate is 0.06.

Further assume that a dividend of amount D is paid to holders of the stock at a date six months into the life of the option.

What is the value of the option if:
(i) D = 0

Answer: No Dividend (D=0)

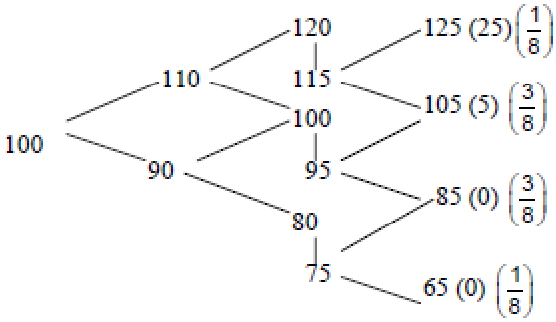


Value of call option:

$$c = \left[(30)\left(\frac{1}{8}\right)+(10)\left(\frac{3}{8}\right)\right]\exp(-0.06*1) = 7.063$$

(ii) D = 5

Answer: Dividend of 5 (D=5) received after 6 months.



The Dividend of 5 received after 6 months has the effect of reducing the stock price by 5 after the dividend is received. In the diagram, we reduce the 9- month prices by 5. This means that the pay-offs are reduced. Hence the value of the call option is lower as a result of the dividend:

$$c = \left[(20)\left(\frac{1}{8}\right) + (5)\left(\frac{3}{8}\right) \right] \exp(-0.06*1) = 4.709$$

(iii) D = 10

Answer: When the dividend is 10, the value of the call option falls again:

$$c = \left[(20)\left(\frac{1}{8}\right) \right] \exp(-0.06*1) = 2.354$$

Credit Risk

Question 1

What is 1-year hazard rate for C grade bonds for 4th year if average cumulative default rate for year 3 is 22.753 and for year 4 is 26.05?

Answer:

Unconditional probability of default in year 4:

V(3) – V(4) = 100 – 22.753 – (100 – 26.05) = 3.297 (%).

The cumulative probability that the bond will survive until end of year 3:

V(2) = 100 – 22.753= 77.247 (%).

The hazard rate at year 4:

$\lambda(4) = 3.297/ 77.247 = 0.0427$

Question 2

To fund an expansion in its operations Company X has just issued 5-year zero coupon bonds with a total face value of 10 million, taking its total asset value to 15 million.

(a) Explain how the value of the bonds can be expressed in terms of a European put option.

Answer:

This is a Merton model.

After the issue of the bond the total current value of the company will be V(0)=15 million. In 5 years' time the company will have an unknown value of V(5).

The bondholders have the first call on the company's assets at that time, so they will receive 10 if F(5)>10 or F(5) otherwise: min[F(5),10]=10-max[10-F(5),0]. The function max[10-f(5),0] is the payoff for a European put option on F(t) maturing at time 5 with strike price 10.

(b) Hence calculate the fair price of a holding of company bonds with face value of 100 using Black-Scholes model, given that the price of a 5-year zero-coupon government bond is 77.88. Assume that the annualized volatility of the company's asset's over 5-year period is 25%.

Answer:

F(0)= S0=15, K=10, T-t=5, σ=0.25

The risk-free interest rate:

100exp(-5*r)=77.88 => r=0.05

d_1= 1.45204

d_2=0.89302

$p=10\exp(-0.25)\Phi(-0.89302)-15\Phi(-1.45204)=7.788*0.18592-15*0.07325=0.349$

So the total value of the bonds now is:

Kexp(-r*(T-t))-p=7.788-0.349=7.439 => 7.439 (million)

So the fair price is 74.39 per 100 face value.

(c) Explain what is meant by credit spread and calculate its value for the company bonds.

Answer:
The price of Company X bond is less than the price of government bonds (77.88) because of the risk of default. As a result r_B will be slightly higher than r (5%):
100exp(-5*r_B)=74.39 => r_B=0.0592
The difference between the bond yield of 5.92% and the default-free rate 5%, i.e. 0.92% (per annum continuously compounded) is called credit spread.

<u>Question 3</u>
Company Y has just issued 4-year zero-coupon bonds with a nominal value of £4 million. The total value of Company Y now stands at £5.5 million. A constant (continuously-compounding) risk-free rate of return is 3% pa.

(a) Use the Merton model to calculate the theoretical price of £100 nominal of Company's Y bonds (assuming 35% of the annual volatility of the value of the company's assets).

Answer:
Under Merton model shareholders in the company receive a payoff after 4 years equivalent to that from a call option with strike price equal to the amount to be repaid to the bondholders.
Use BS formula for call option to assess the current value of a shareholding:

E(0) = 5.5 Φ(d1) - 4*exp (-0.03*4) Φ(d2)

where d1=({ln(5.5/4)+(0.03+(0.35^2)/2)*4}/0.35*4^0.5)=3.90545
and d2=d1-0.35*4^0.5=3.20545

E(0) =5.5 Φ(3.90545) - 4*exp (-0.03*4) Φ(3.20545) = 1.954452

Thus the value of the bond at t0 is
B(0) = 5.5 – 1.954452 = 3.545548

So the theoretical price of 100GBP nominal of these bonds is:
(3.545548/4)*100 = 88.6387

(b) Estimate the risk-neutral probability of default on Company's Y bonds.

Answer:
Under Merton model the probability of default is:
1-Φ(d2)=1-0.999326=0.000674